Solution: Conceptually Confused !

Category : Physics + Electronics

<u>This problem is mainly based on the concept of '*Fluids*'</u>

In order to solve the problem, we first need to understand the setup and what is going to exactly happen. There are some important observations to make before proceeding !

1. **<u>In Fig.1, Radius of the base for all the 3 containers is same.</u> (——)**

- This implies that the base area for all the 3 containers is same

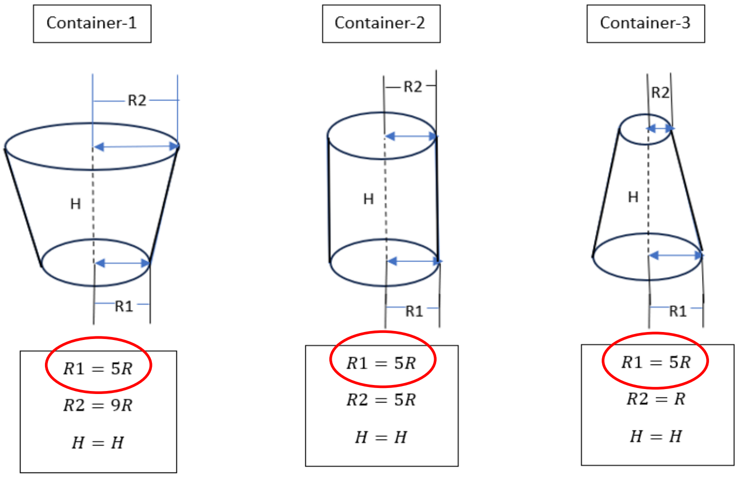
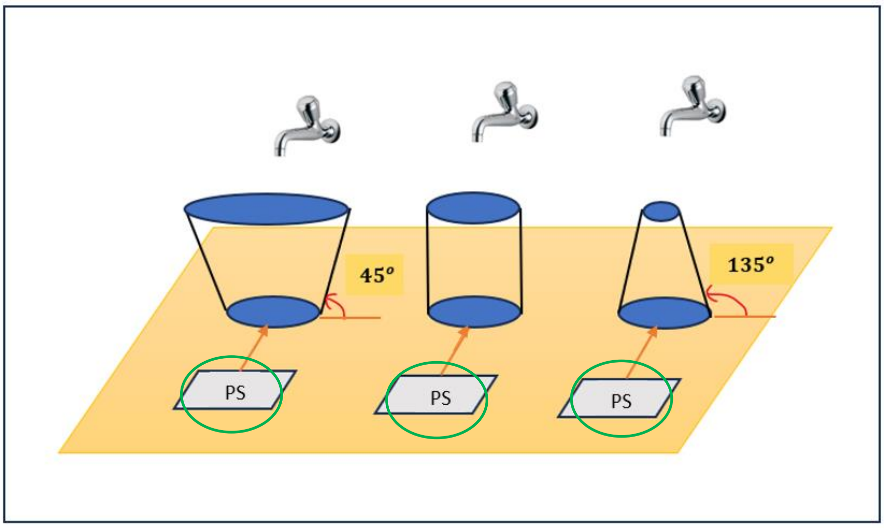


Fig.1 Dimensions of all the vessels

2. **<u>In Fig.2, Pressure sensors are attached to the base of the containers</u> (——)**

- This means that, we need to be interested in the pressure at the bottom of the containers

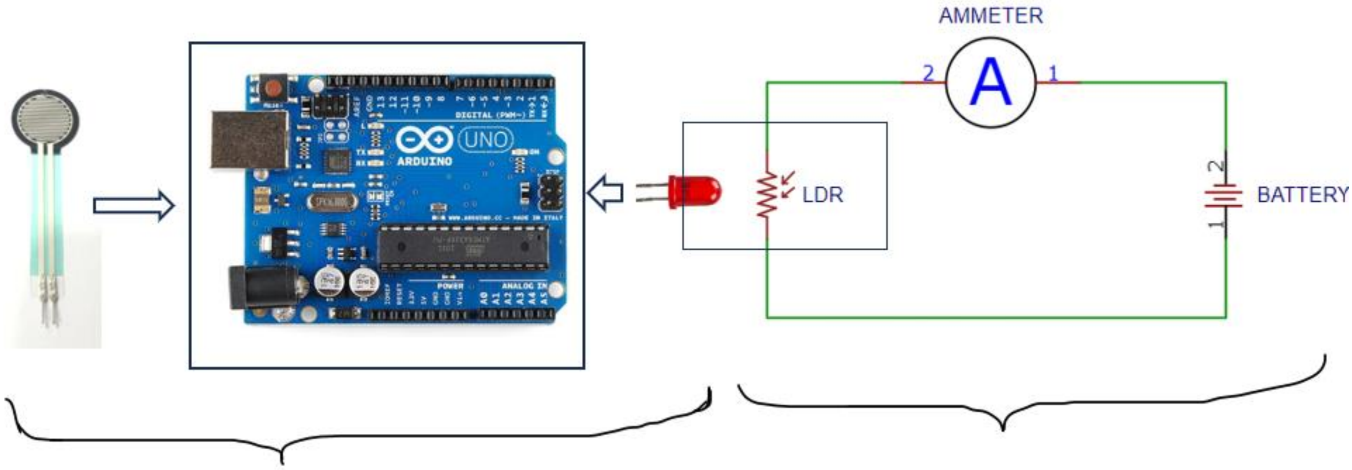


3. The Pressure sensor is connected to the circuit as shown in Fig.3. So basically, based on the pressure values, we will be controlling some other things. What exactly happens, is decided by the Arduino IDE code.

About Photoresistor

- More Light → Less Resistance
- Less Light → More Resistance

Simplified version of Fig.3 given in Question



Connections 1 : Force sensor and LED connected to Arduino

Connections 2 : A separate circuit involving Ammeter, LDR and Battery. The value of LDR is controlled by LED

Fig.3

- Basic overview of what actually happens (Gist of the code) :

Pressure at the base is measured by the pressure sensor

Based on the pressure measurements, the sensor returns some values (more the pressure → more the value)

Those values are used by the microcontroller to control the brightness of LED

The light emitted by this LED falls on the photoresistor.

More the resistance value, lesser current will be circulated in the circuit

This current value is measured by ammeter

Flowchart - 1

Solution to Question Parts :

Part – (a)

The variation of the height of water column in these 3 containers is given w.r.t time in the form of graphs. One simple observation is that, the containers are empty at t=0 (since h=0 for all)

We know that the pressure on base is 'only' dependent on the height of the water column above it (base) . This can be verifies by the relation :

$$\boxed{P = P_o + \rho g h}$$

Note : But how is this possible ? How can it only depend upon height as it is clearly visible that for same height, the volume of the containers is very different !!

- This is called as '*Hydrostatic Paradox*'

Explanation :

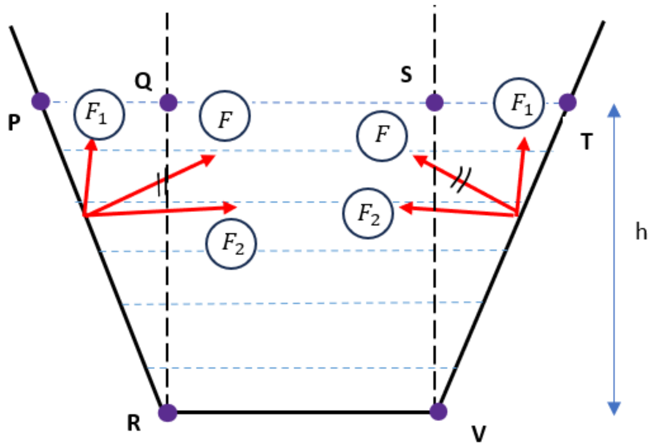


Fig. 4

- The F_2 component cancels out each other while the F_1 component adds up in vertical direction. (This was just for one force F given by container on water). There will be many such forces.

- After summing up all the components, we can see that the horizontal components get cancelled out with each other but the vertical components keep adding up

- This total vertical component = wt of water in region ΔPQR + wt of water in region ΔSTV

So, finally, we just have the effect of cylindrical region acting on the base of the container.

(Do the same analysis for container 3)

So the whole value of pressure just depends upon height here. So, let's plot all the curves on a single graph in order to get an idea of what is exactly happening ?

All plots on single graph :

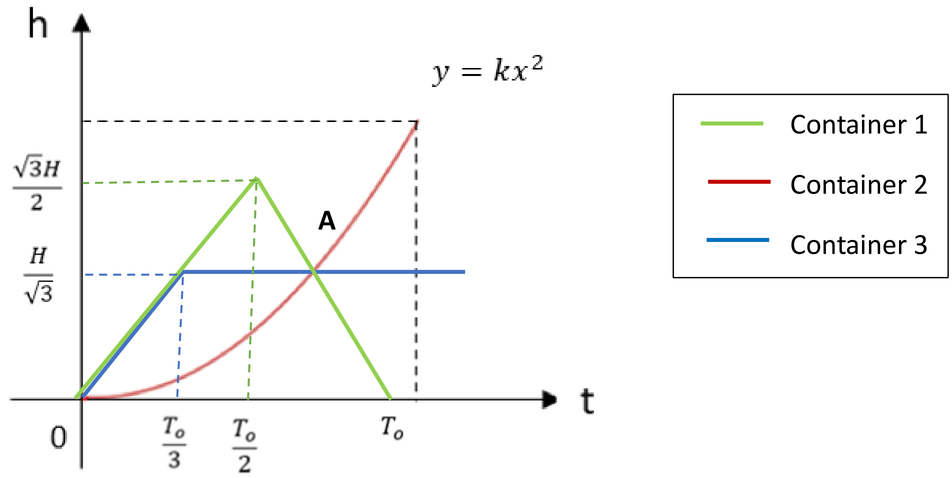
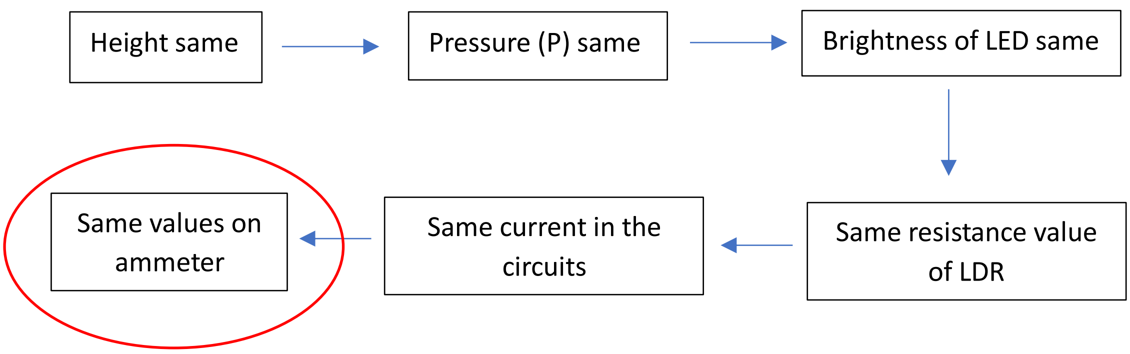


Fig. 5

Now, wherever there are intersections, it implies that the height of water columns in those containers is same at that instant. So, the flowchart goes like :



Flowchart – 2

- From the above graph, we can see that, in region: $0 < t < \frac{T_o}{3}$, the height of water column in both, container 1 & 3 increases at same rate. And, as discussed in Flowchart-2 , this will cause the ammeters in circuit 1 and 3 to show same values.

- But there is one more point (point A), where all the 3 plots intersect. This means that, at that instant, height of water column in all the 3 containers was same. So, this will make the ammeters in circuit 1, 2 and 3 to indicate same values

Finding point A :

Solving for $h = (-\frac{\sqrt{3}H}{T_o})(t - T_o)$ & $h = \frac{H}{\sqrt{3}}$,

We get the point A as $\left(\frac{2T_o}{3}, \frac{H}{\sqrt{3}}\right)$. We see if point A is satisfied by curve of container 2 or not. (i.e. $h = kt^2$)

LHS = h = $\frac{H}{\sqrt{3}}$

RHS = $kt^2 = \frac{9H}{4\sqrt{3}\,{T_o}^2}(\frac{2T_o}{3})^2 = \frac{H}{\sqrt{3}}$ = LHS……… (Hence, it satisfies → ∴ Verified)

Part – (b)

In this part, we are just asked to make the comparisons of the current values shown in the ammeters of the 3 circuits

- Let us call the currents as i1, i2 and i3 flowing in circuits attached to container 1, 2 and 3 respectively.

- For interval 1 : i1 = i3 > i2
- For interval 2 : i1 > i3 > i2

- For interval 3, it's not the same relation throughout. So we divide the interval 3 into two further subintervals 3.1 and 3.2

Interval 3.1 : $(\frac{T_o}{2}, \frac{2T_o}{3})$

Current Relation : i1 > i3 > i2

Interval 3.2 : $(\frac{2T_o}{3}, T_o)$

Current Relation : i2 > i3 > i1

Part – (c)

Refer Fig.3 and Flowchart – 1 in this solution.
