Solution : Vectors on board

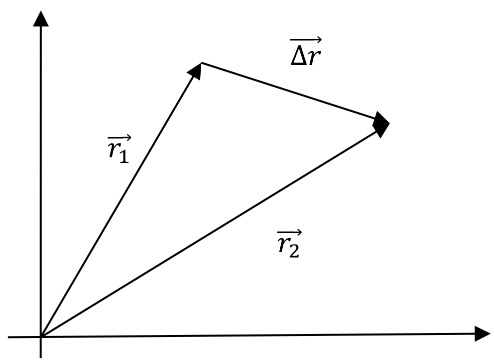
Chess OR Physics? -2 - Saurabh Salvi

The problem is purely based on the concepts of 'Vectors'.

Concept :

Displacement vector : The vector which represents the change in position vector for a body pointing from initial position to final vector is basically known as displacement vector.

Diagram :



Here,

$\overrightarrow{\Delta r}$ (or say $\overrightarrow{r_3}$) = $\overrightarrow{r_2} - \overrightarrow{r_1}$

Where,

$\overrightarrow{r_1}$ is the position vector for initial position of body

$\overrightarrow{r_2}$ is the position vector for final position of body

$\overrightarrow{r_3}$ is called displacement vector since it represents the displacement (vector quantity) of the body.

Returning to our problem solution,

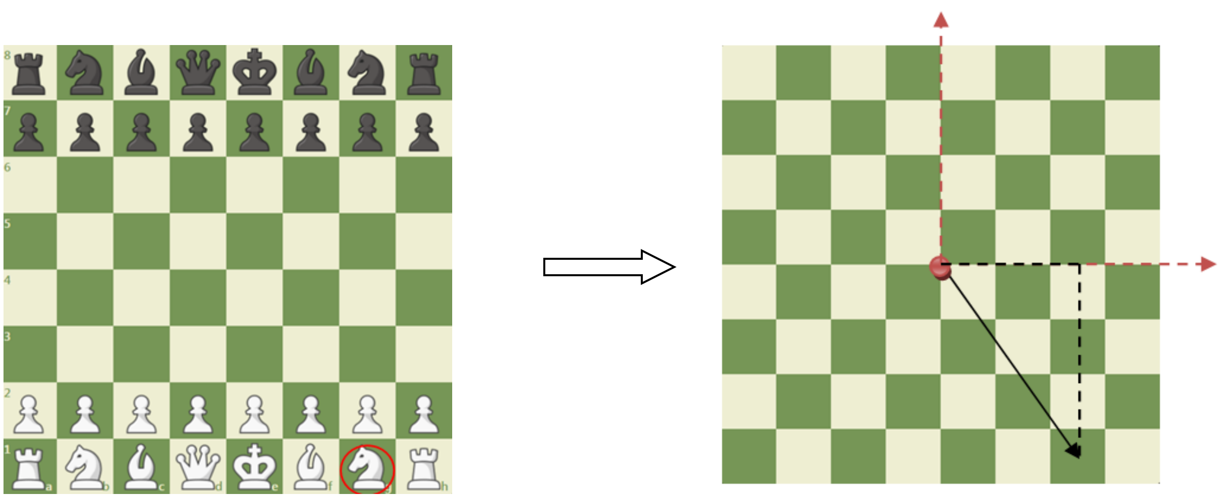
There are two ways through which we can approach the problem and both are necessary for our better understanding :

(i) Quick method ***(ii) Polygon law of vector addition.***

(i) Quick method :

Since the quantity here we are asked to deal with is 'displacement' and displacement being a vector quantity, we need to be just concerned about the initial and the position of the body.

Consider the geometric center of the board as origin(●) and we define X and Y axes (- - - - -) (i.e. the 2-D coordinate system) as shown.

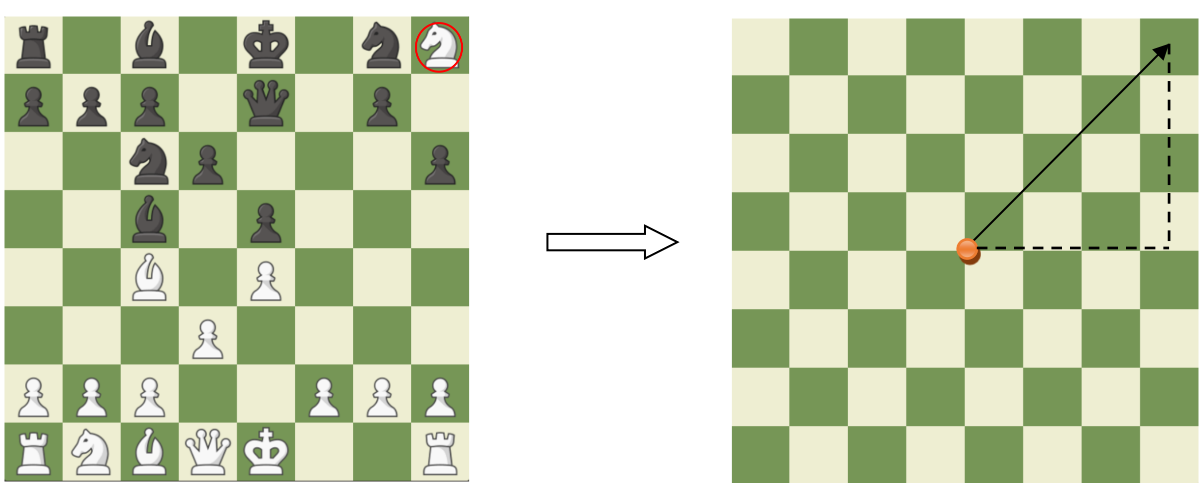


Initial position

So the initial vector can be written as (say $\vec{r_1}$), then

$$\vec{r_1} = (5L/2)i + (-7L/2)j$$

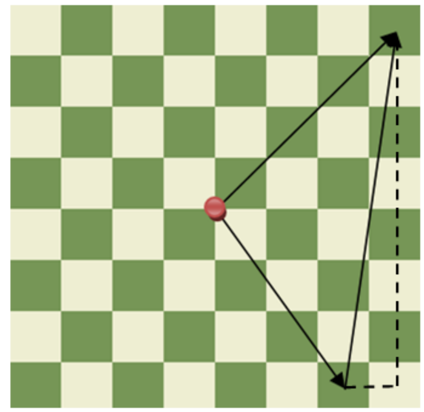
$$\vec{r_1} = (5L/2)i - (7L/2)j \quad \text{------(1)}$$



Here, for the final position, we can write the final position vector ($\vec{r_2}$) as :

$$\vec{r_2} = (7L/2)i + (7L/2)j \quad \text{-----(2)}$$

From triangle law of vector addition in adjacent figure,



$$\vec{r_2} = \vec{r_1} + \vec{r_3}$$

$$\therefore \quad \vec{r_3} = \vec{r_2} - \vec{r_1}$$

$$= [\,(7L/2)i + (7L/2)j\,] - [\,(5L/2)i - (7L/2)j\,]$$
....(from (1) and (2))

$$\vec{r_3} = (L)i + (7L)j \quad \text{-----(3)}$$

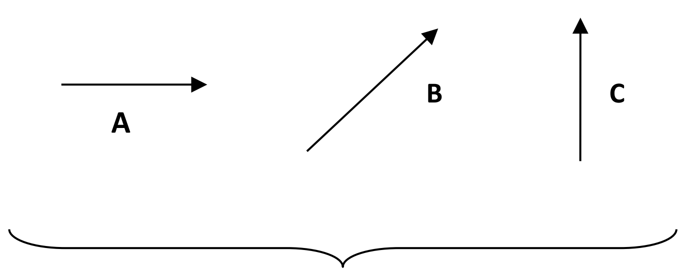
The eqn(3) can even be verified from the adjoint figure as well !!

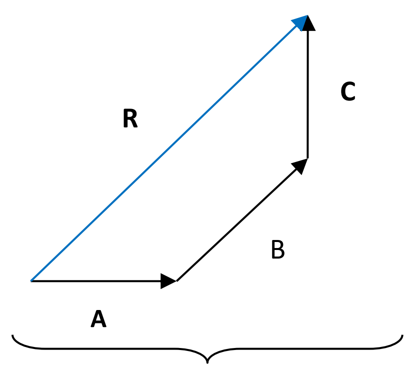
So, **the total displacement vector for the g1 knight was** $\boldsymbol{\vec{r_3} = (L)i + (7L)j}$

(ii) Polygon Law of vector addition :

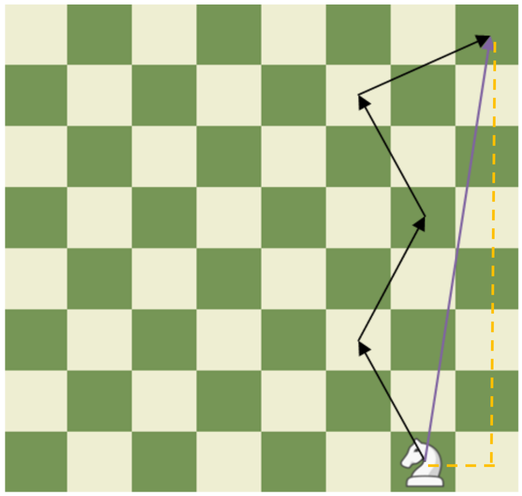
It helps in addition of two or more vectors. For e.g. if we have three vectors $\vec{A}, \vec{B}, \vec{C}$ (**A**, **B**, **C**)and our aim is to find the resultant of the these vectors ($\vec{R}$),then we need to first arrange the vectors in order (i.e. join the tail of **B** to head of **A** & tail of **C** to head to **B**) and then we get the resultant by joining the tail of the first vector (here **A**) to the head of the last vector (here **C**)

R = A + B +C





In the game, white's g1 knight did move multiple times to various squares (from g1 to f3, then from f3 to g5, then from g5 to f7 and then finally from f7 to h8). These moves can be represented as displacement vectors and the resultant of which will give the total displacement of the g1 knight from initial position to final position which will be our answer.



Moves of knight	Corresponding vectors
g1 to f3	(-L)i + (2L)j
f3 to g5	(L)i + (2L)j
g5 to f7	(-L)i + (2L)j
f7 to h8	(2L)i + (L)j

Now for the total displacement vector, we need to just add all the corresponding vectors which results as:

Total displacement vector = [(-L)i + (2L)j] + [(L)i + (2L)j] + [(-L)i + (2L)j] + [(2L)i + (L)j]

= (L)i + (7L)j

This can be verified from the above figure as well

So, **the total displacement vector for the g1 knight was $\vec{r_3}$ = (L)i + (7L)j**

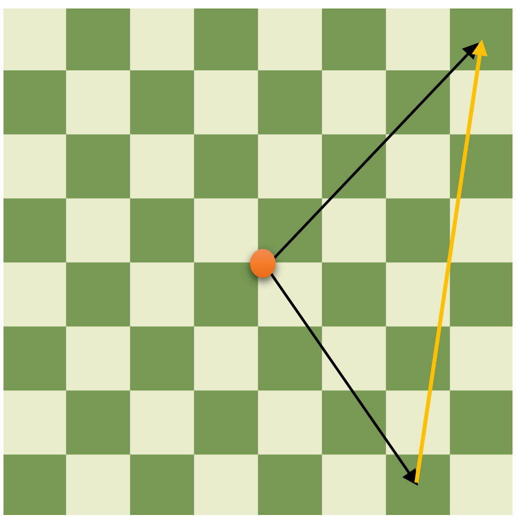
Part (b) :

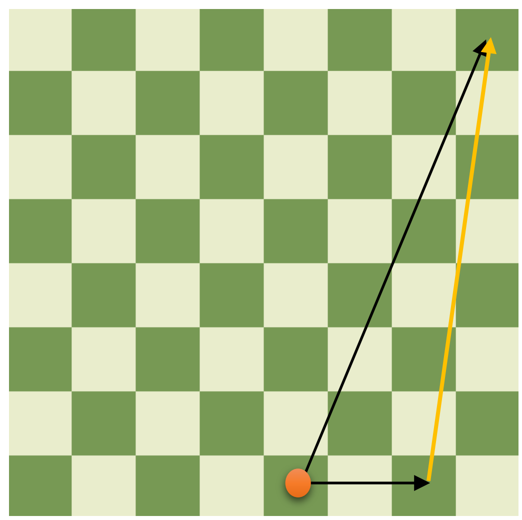
Regarding problem :

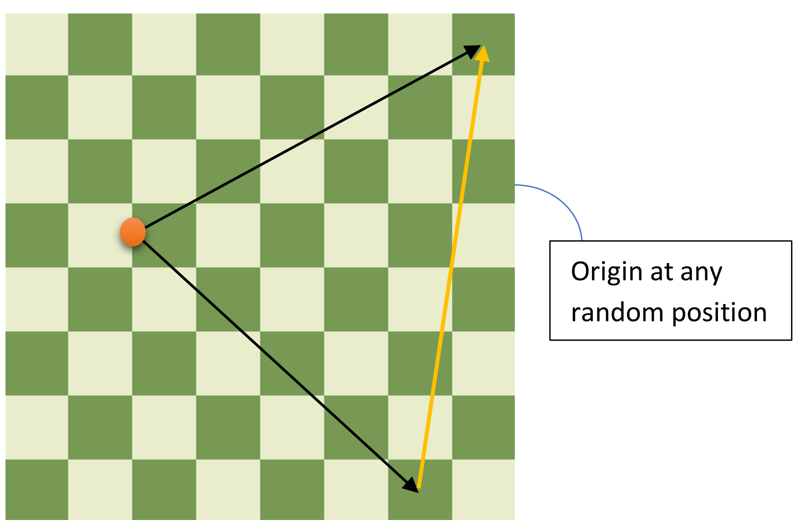
- In the whole problem, we considered the geometric center of the board to be the origin but you can use any point in 2-D plane as your origin and even with that, you are bound to get the same answer for the total displacement vector. The reason behind this being that the displacement vector **literally just doesn't care** about the origin. The main thing for it is concerned is the initial position of body and the final position to which body goes.

Compare the following cases and just very by yourself that the displacement vector is independent of the origin considered. (observe yellow vector (displacement vector))

''Position vectors change with origin but displacement vector do not''







Regarding chess *:*

- The opening played in the game is the '*Italian opening*'. It proceeds as (1. e4 e5 2. Nf3 Nc6 3.Bc4)

- The move Nxf7 (check question) was 'game-deciding' because it **forks** (i.e. attacks two major pieces at the same time) Queen and the rook, which means trying to save any one of them would result in capture of other. This is one of the most common trap in Italian for which most of the beginners fall. *So beware!!*