

Solution : Smartwork > Hardwork

Given :

- i. Solid angle for all discs is Ω . The relation between the semi vertex angle of the cone formed and the solid angle can be given as:

$$\Omega = 2\pi(1 - \cos\theta)$$

- ii. In the relation $x = nx_0$, n actually represents the disc number

i.e. $x = x_0$ occurs at $n=1$, so first disc

$x = 2x_0$ occurs at $n=2$, so second disc

and so on..

- iii. By (II) relation,

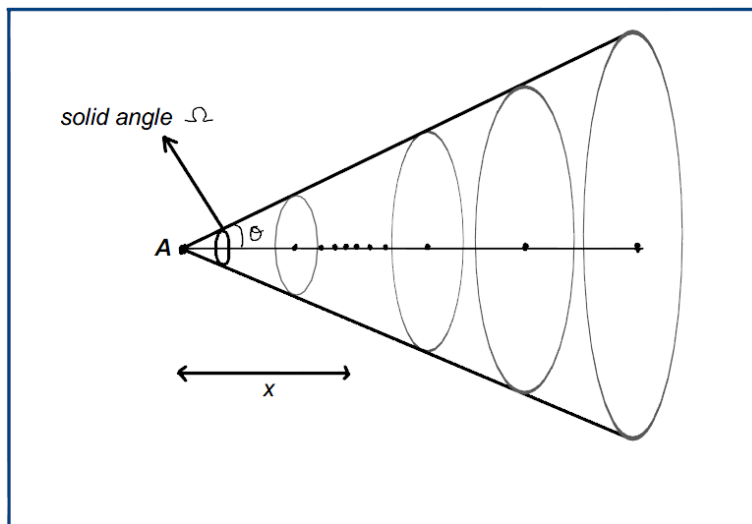
When $n=1$ (first disc), $\sigma = \sigma_0$

When $n=2$ (first disc), $\sigma = -\sigma_0$

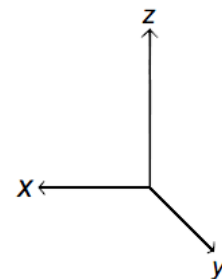
When $n=3$ (first disc), $\sigma = \sigma_0$

and so on..

The arrangement can be shown as :



Figure



(a) We are given that when $x = x_0$, the radius for that disc is $2R_0$

From the graph we can derive the relation as follows :

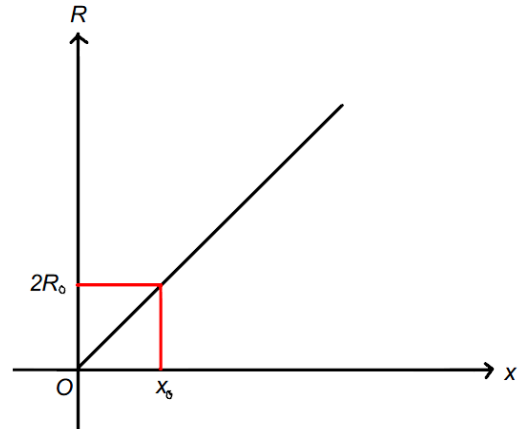
Slope of the line can be given as:

$$m = \frac{2R_0 - 0}{x_0 - 0}$$

$$m = \frac{2R_0}{x_0}$$

The relation can be given by

$$R = \left(\frac{2R_0}{x_0}\right)x$$



(b) So, here we have classified people as Hardworking & Smartworking based on the approach they select to solve the problem.

From the function of radius derived in part (a) , and keeping the radius of the 'last' disc ($42R_0$) in it, we get the total number of discs present in the arrangement.

We have, $R = \left(\frac{2R_0}{x_0}\right)x$

$$42R_0 = \left(\frac{2x}{x_0}\right)R_0$$
$$x = 21x_0$$

On comparing with (I) ($x = nx_0$)

$$n = 21$$

→ So, we have 21 discs in the system

(Electric field Intensity at A $\rightarrow E_A$)

Hardworking people	Smartworking people
<div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> $E_A = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{R^2 + x^2}} \right]$ </div> <p>To find E_A, they will straight away start applying the formula for each disc one by one since now for each disc, x and R is known.</p>	<div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> $E_A = \frac{\sigma}{2\epsilon_0} (1 - \cos\theta)$ </div> <p>A nice observation can be made when it is noticed that, the angle subtended at A is same for all discs and hence E_A magnitude will be same by each disc.</p>
<p>Lengthy to solve</p>	<p>Since the discs are alternately oppositely charged (refer question), electric field due to first 20 plates combined will get zero.</p> <p>For $+\sigma_0$, E will be along + X axis (\hat{i}) For $-\sigma_0$, E will be along - X axis ($-\hat{i}$)</p> <p>Therefore, the 21st plate (positively charged ($n=21$)) is overall responsible for the field at A.</p>

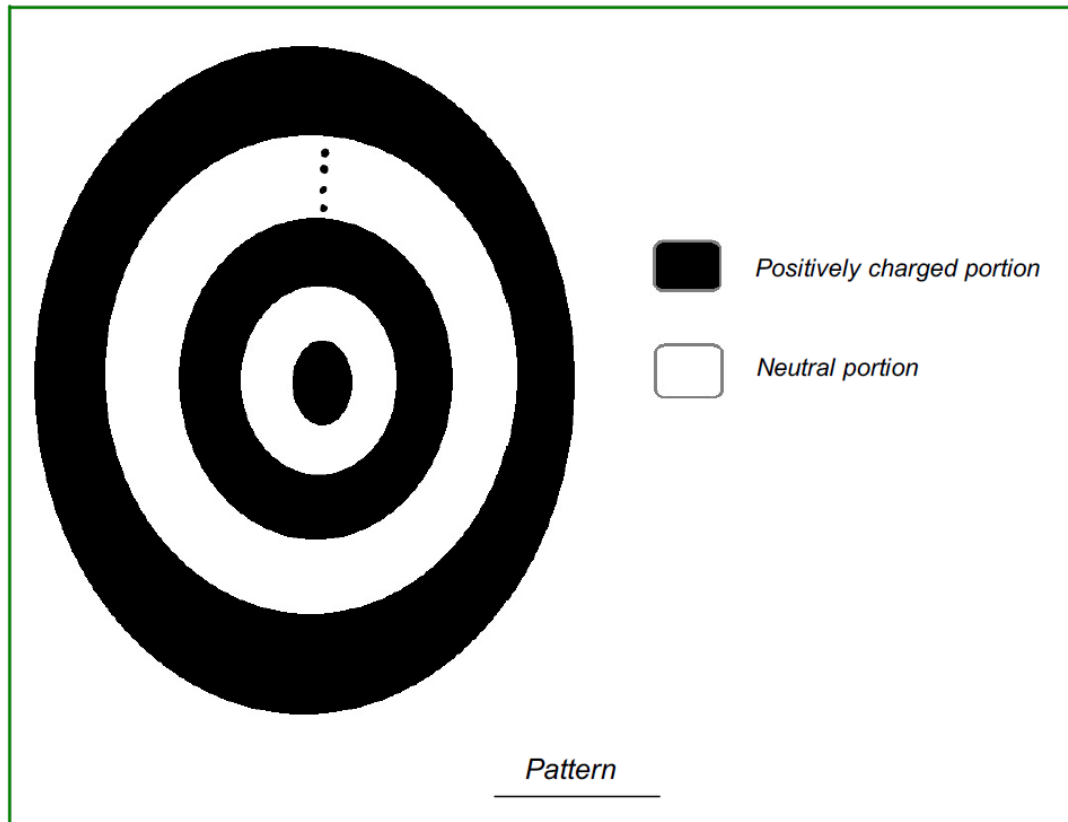
Answer :

Electric field at A is given by :

$$\vec{E}_A = \frac{\sigma_0 \Omega}{4\pi\epsilon_0} \hat{i}$$

(c) When all the discs are merged, since the surface charge density is alternately varying by sign, some part becomes neutral, some remain positively charged. And when the black coloured negatively charged ink is spread on the pattern, it gets attached to positively charged portion.

The pattern formed appears like below :



**Important aspects of the problem are highlighted*